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CAN THE CURVATURE OF AN OPTICAL GLASS FIBER BE DIFFERENT FROM THE CURVATURE OF ITS COATING?

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Abstract—The study is aimed at the evaluation of the relationship between the measured (or imposed) constant curvature of the coating of an optical glass fiber and the elastic curve of the fiber itself. It is shown that the buffering effect of the coating is different for different points along the curved area and depends on the length of this area, and the compliance of the coating. In the case of a very short curved area and/or a very compliant coating, the curvature of the glass fiber is smaller than the curvature of the coating and increases with an increase in the length of the curved area and the coating stiffness. In the case of a long curved area and/or a stiff coating, both curvatures are practically the same for almost the entire curved area. Only when approaching the ends of this area, the ratio of the curvature of the glass fiber to the coating curvature somewhat increases (by a factor of 1.043) and then rapidly drops to unity at the ends. There are, however, some "intermediate" unfavorable combinations of the lengths of the curved area and coating compliances that result in curvature ratios exceeding (by up to a factor of 1.086) the coating curvature in the midportion of the curved area. It is shown that such a paradoxical situation is due to the redistribution of the interfacial radial load at certain combinations of the lengths of the curved areas and spring constant of the coating. For a current AT&T dual-coated fiber design with a 30 μm thick silicone primary coating, the curvature ratio is greater than unity when the lengths of the curved area fall within the range between 1.84 and 4.27 mm, and reaches the 1.086 value when the length of curved area is about 2.44 mm.

INTRODUCTION

Clearly, the curvature of an optical glass fiber in a glass-coating composite can be different from the measured (or imposed) curvature of its coating. This is due to the buffering effect of the coating material. Intuitively it is felt that such an effect can be essential in short fibers (or in fibers with short curved portions) especially with compliant (low modulus) coatings. If this effect is appreciable, it should be accounted for in those cases, when the actual local curvature of the glass fiber is important, but cannot be determined by direct measurements. Frequently, information on fiber curvature is needed to assess the added transmission losses and/or the level of bending stress in the glass fiber.

The present analysis is aimed at the evaluation of the relationship between the curvature of the glass fiber and the curvature of the coating. The investigation is based on an analytical stress model in which the glass fiber is treated as a beam lying on an elastic foundation [see, for instance, Timoshenko and Young (1965)], provided by the coating material.

MAJOR ASSUMPTIONS

The following major assumptions are used in this analysis:

(1) The curvatures of the glass fiber and its coating are small enough, so that the linear theory of bending of beams [see, for instance, Timoshenko and Young (1965)] can be applied.

(2) The deflections of the glass fiber can be evaluated from the radial (normal) interfacial stresses only, without considering the interfacial shearing stresses. As is known [see, for instance, Suhir (1989)], both radial (normal) and axial (shearing) interfacial stresses arise in a bi-material composite structure subjected to thermally induced or external loading. However, the curvatures of the components of such a composite are affected by the normal stresses to a significantly greater extent than by the shearing stresses. Therefore, having in

mind that this analysis is aimed at the evaluation of curvatures rather than stresses, the above assumption is thought to be valid.

(3) The compatibility condition for the deflections can be written as

$$p(x) = K[w_c(x) - w_0(x)], \tag{1}$$

where $p(x)$ is the radial (normal) loading, $w_c(x)$ and $w_0(x)$ are the deflection functions of the coating and the glass fiber, respectively, and K is the spring constant of the coating. The relationship (1) states that the loading acting on the glass fiber is proportional to the difference in the deflections of the coating and the fiber itself. This relationship uses an obvious assumption that the load $p(x)$ at the given x can be evaluated on the basis of a "two-dimensional approach", i.e. depends on the deflections in the given cross-section only, and is not affected by the strains and stresses in the adjacent cross-sections.

(4) The spring constant K remains unchanged throughout the entire curved area and can be computed on the basis of a solution to a corresponding two-dimensional (plane strain) problem of the theory of elasticity (Vangheluwe, 1984):

$$K = \frac{4\pi E_1(1-\nu_1)(3-4\nu_1)}{(1+\nu_1)\left[(3-4\nu_1)^2 \ln\frac{r_1}{r_0} - \frac{(r_1/r_0)^2 - 1}{(r_1/r_0)^2 + 1}\right]}. \tag{2}$$

Here r_0 is the glass fiber radius, r_1 is the radius of the (primary) coating, and E_1 and ν_1 are the elastic constants of the coating material. This formula has been obtained for a dual-coating system, assuming absolutely rigid secondary coating, and therefore the coating compliance is due to the primary coating only. If this formula is applied to single-coated fibers it is thought to result in a reasonable overestimation of the spring constant. Such an overestimation is associated with the fact that an assumption of the nondeformability of the outer contour of the (primary) coating, which underlies eqn (2), is justified for a single coating system to a lesser extent than for a dual-coated design. We would like to point out that if the compliance of the secondary coating is not negligibly small compared to the compliance of the primary coating, then a more complicated formula, accounting for the finite compliance of the secondary coating, should be applied (Suhir, 1988a).

ANALYSIS

Basic equation and boundary conditions

Let a portion of a glass-coating composite be bent on a circular mandrel, so that the radius of curvature R_c of the coating is constant. Because of the finite compliance of the coating, the radius $R_0(x)$ of curvature of the glass fiber can be different from the radius R_c (Fig. 1). This results in the radial interfacial forces $p(x)$ which must satisfy the following equation of equilibrium:

$$\int_{-l}^{x}\int_{-l}^{\xi} p(\xi')\,d\xi'\,d\xi = \frac{E_0 I_0}{R_0(x)}. \tag{3}$$

Here l is half the length of the curved area, $E_0 I_0$ is the flexual rigidity of the glass fiber, E_0 is Young's modulus of glass, $I_0 = (\pi/4)r_0^4$ is the moment of inertia of the glass fiber cross-section, and r_0 is its radius. The origin is in the middle of the curved area on the centerline of the coating. Equation (3) states that the external bending moment due to the load $p(x)$ must be equilibrated by the elastic bending moment acting over the cross-sections of the glass fiber.

The radii of curvature of the glass fiber and its coating can be expressed through the second derivatives of the deflection functions as

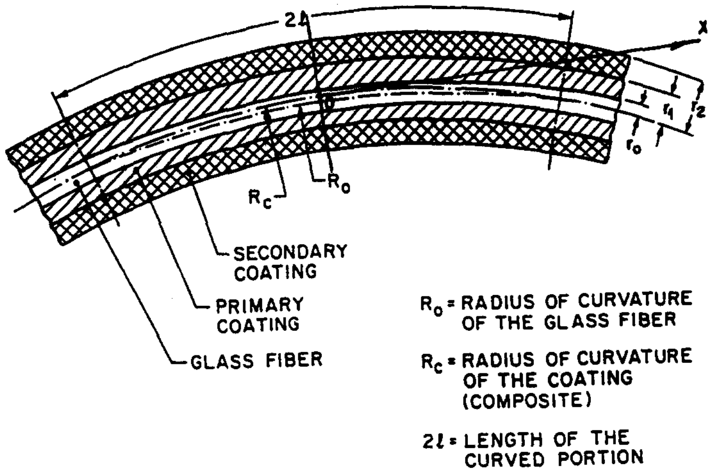

Fig. 1. Coated optical fiber subjected to bending.

$$R_0(x) \cong \frac{1}{w_0''(x)}, \quad R_c \cong \frac{1}{w_c''(x)}.$$

Then eqn (1) can be written as follows:

$$\frac{1}{R_0(x)} = \frac{1}{R_c} - \frac{p''(x)}{K}. \tag{4}$$

Substituting this relationship into the equilibrium condition (3), we obtain the following integral equation for the unknown load function $p(x)$:

$$p''(x) + 4\alpha^4 \int_{-l}^{x} \int_{-l}^{\xi} p(\xi')\,d\xi'\,d\xi = \frac{K}{R_c}, \tag{5}$$

where

$$\alpha = \sqrt[4]{\frac{K}{4E_0 I_0}} = \frac{1}{r_0}\sqrt[4]{\frac{K}{\pi E_0}} \tag{6}$$

is the parameter of the coating compliance. From (5), by differentiation, we find:

$$p'''(x) + 4\alpha^4 \int_{-l}^{x} p(\xi)\,d\xi = 0, \tag{7}$$

$$p^{\mathrm{IV}}(x) + 4\alpha^4 p(x) = 0. \tag{8}$$

Since no other loads, except $p(x)$, act on the glass fiber, this load must be self-equilibrated, i.e. the following conditions are to be fulfilled:

$$\int_{-l}^{l} p(x)\,dx = 0, \quad \int_{-l}^{l} \int_{-l}^{\xi} p(\xi')\,d\xi'\,d\xi = 0. \tag{9}$$

These conditions reflect the requirement that both the shear force and the bending moment at the end $x = l$ must be zero. Applying the first of the conditions (9) to eqn (7) and the second of these conditions to eqn (5), we conclude that the function $p(x)$ must satisfy the following boundary conditions:

$$p'''(l) = 0, \quad p''(l) = \frac{K}{R_c}. \tag{10}$$

Interfacial radial load

The solution to eqn (8), satisfying the conditions (10), can be sought in the form:

$$p(x) = p_0\left[V_0(\alpha x) - \frac{V_1(u)}{V_3(u)} V_2(\alpha x)\right], \tag{11}$$

where

$$p_0 = p(0) = -\frac{Kl^2}{6R_c}\chi_1(u) \tag{12}$$

is the load per unit fiber length in the middle of the curved area, the parameter u is expressed as

$$u = \alpha l = l\sqrt[4]{\frac{K}{4E_0 I_0}} = \frac{l}{r_0}\sqrt[4]{\frac{K}{\pi E_0}}, \tag{13}$$

and the functions entering the formulae (11) and (12) are

$$\left.\begin{aligned} V_0(\alpha x) &= \cosh \alpha x \cos \alpha x \\ V_1(\alpha x) &= \frac{1}{\sqrt{2}}(\cosh \alpha x \sin \alpha x + \sinh \alpha x \cos \alpha x) \\ V_2(\alpha x) &= \sinh \alpha x \sin \alpha x \\ V_3(\alpha x) &= \frac{1}{\sqrt{2}}(\cosh \alpha x \sin \alpha x - \sinh \alpha x \cos \alpha x) \end{aligned}\right\}, \tag{14}$$

$$\begin{aligned} \chi_1(u) &= \frac{3}{u^2}\frac{V_3(u)}{V_0(u)V_1(u) + V_2(u)V_3(u)} \\ &= \frac{6}{u^2}\frac{\cosh u \sin u - \sinh u \cos u}{\sinh 2u + \sin 2u}. \end{aligned} \tag{15}$$

The functions (14) obey the following simple rules of differentiation:

$$\left.\begin{aligned} V_0'(\alpha x) &= -\alpha\sqrt{2}V_3(\alpha x), & V_1'(\alpha x) &= \alpha\sqrt{2}V_0(\alpha x) \\ V_2'(\alpha x) &= \alpha\sqrt{2}V_1(\alpha x), & V_3'(\alpha x) &= \alpha\sqrt{2}V_2(\alpha x) \end{aligned}\right\}, \tag{16}$$

which make the utilization of these functions of convenience. Obviously,

$$V_0(0) = 1, \quad V_1(0) = V_2(0) = V_3(0) = 0.$$

The function (15) is introduced in such a way that it is equal to unity in the case of an ideally compliant coating ($K = 0$) and tends to zero for an absolutely stiff coating ($K \to \infty$).

The load at the ends of the curved area is

$$p_l = p(l) = \frac{Kl^2}{3R_c}\chi_2(u), \tag{17}$$

where the function

$$\chi_2(u) = \frac{3}{2u^2}\frac{\sinh 2u - \sin 2u}{\sinh 2u + \sin 2u} \tag{18}$$

is also equal to unity for $K = 0$ and tends to zero for $K \to \infty$. The tabulated values of the functions $\chi_1(u)$ and $\chi_2(u)$ are given in Table 1.

In the case of short curved areas and/or very compliant coatings ($u < 1$) the formulae (11), (12) and (17) yield:

$$p(x) = p_0\left(1 - 3\frac{x^2}{l^2}\right), \quad p_0 = -\frac{Kl^2}{6R_c}, \quad p_l = \frac{Kl^2}{3R_c}. \tag{19}$$

Hence, in this case the load function $p(x)$ is expressed by a parabola and therefore has a single internal extremum (minimum) at the origin.

In order to find out whether, if the u value is not small, additional internal extrema can occur on the curve $p(x)$, we form, in accordance with the well-known procedure [see, for instance, Arfken (1985)] an equation $p'(x) = 0$. This results in the following equation for the locations $x = x_*$ of the expected extrema:

$$V_1(u)V_2(ax_*) - V_3(u)V_0(\alpha x_*) = 0,$$

or

Table 1

μ	$\phi_1(\mu)$	$\chi_1(\mu)$	$\chi_2(\mu)$
0	1.000	1.000	1.000
0.1	1.000	1.000	1.000
0.2	1.000	1.000	1.000
0.3	0.999	0.999	0.999
0.4	0.996	0.996	0.997
0.5	0.990	0.991	0.993
0.6	0.979	0.982	0.985
0.7	0.961	0.967	0.973
0.8	0.935	0.946	0.956
0.9	0.899	0.917	0.931
1.0	0.852	0.878	0.899
1.1	0.795	0.830	0.859
1.2	0.728	0.774	0.813
1.3	0.653	0.712	0.761
1.4	0.573	0.645	0.705
1.5	0.492	0.576	0.648
1.6	0.411	0.509	0.591
1.7	0.335	0.444	0.537
1.8	0.264	0.384	0.483
1.9	0.201	0.328	0.439
2.0	0.144	0.279	0.397
2.2	0.054	0.197	0.325
2.37	0.000	0.147	0.279
2.4	−0.009	0.136	0.269
2.6	−0.051	0.092	0.226
2.8	−0.074	0.060	0.193
3.0	−0.085	0.038	0.167
3.2	−0.087	0.023	0.146
3.4	−0.082	0.012	0.129
3.6	−0.073	0.006	0.115
3.8	−0.063	0.002	0.104
4.0	−0.052	−0.001	0.094
4.2	−0.041	−0.002	0.085
4.4	−0.031	−0.003	0.078
4.6	−0.022	−0.003	0.071
4.8	−0.015	−0.002	0.065
5.0	−0.009	−0.002	0.060

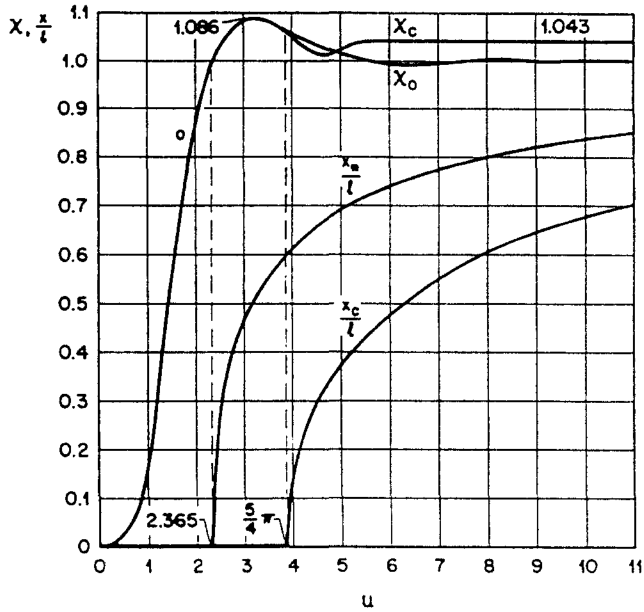

Fig. 2. Ratio of curvature of the glass fiber to the curvature of the coating in the middle of the curved portion of the fiber.

$$\tanh u \tanh\left(u\frac{x_*}{l}\right)+\tan u \tan\left(u\frac{x_*}{l}\right) = 0. \tag{20}$$

The solution to this transcendental equation is plotted in Fig. 2. As is evident from this solution, no additional internal extrema occur, if the u value is below $u_1 = 2.365$, which is the root of the equation

$$\tanh u+\tan u = 0,$$

or

$$\cosh u \sin u+\sinh u \cos u = 0. \tag{21}$$

As has been shown above, the single extremum at the origin in this case is minimum. This conclusion can also be formally made on the basis of the sign of the second derivative $p''(x)$: If this derivative, calculated at $x = x_*$, is positive, the extremum at this location is a minimum; if it is negative, then the function $p(x)$ has a maximum at this location [see, for instance, Arfken (1985)]. With expression (11) for the function $p(x)$, we conclude that if the condition

$$V_1(u)V_0(\alpha x_*)+V_3(u)V_2(\alpha x_*) > 0 \tag{22}$$

is fulfilled, the extremum at $x = x_*$ is a minimum. When $x_* = 0$, this inequality yields $V_1(u) > 0$, or

$$\cosh u \sin u+\sinh u \cos u > 0, \tag{23}$$

and therefore the extremum at the origin is a minimum.

The inequality (23) is violated for u values exceeding $u_1 = 2.365$. This means that the "global" minimum at the origin becomes a local maximum, while two additional extrema occur on both sides of the origin. With the further increase in the u value the local (negative) maximum at the origin decreases in its absolute value, while the absolute values of the new internal minima go up, and these minima shift towards the ends of the curved area (Fig.

2). If, for instance, $u = \pi$, then, in order that the product of the hyperbolic tangents in eqn (21) remains finite, one should put $\tan(u(x_*/l))$ equal to infinity. Then we have $u(x_*/l) = \pi/2$, so that $x_* = (l/2)$. With $u = \pi$ and $\alpha x_* = \pi/2$, condition (22) is fulfilled, and therefore the extrema at $x_* = l/2$ are, indeed, minima. However, condition (23) is violated, and therefore the extremum at the origin is a local maximum.

For sufficiently large αx_* values, the hyperbolic tangents in eqn (20) are close to unity, and this equation can be simplified as follows

$$\cos\left[u\left(1-\frac{x_*}{l}\right)\right] = 0. \tag{24}$$

Then we obtain:

$$\frac{x_*}{l} = 1-\frac{\pi}{2u}. \tag{25}$$

This approximate formula gives satisfactory results for $u > \pi$.

Curvature ratio

The ratio $\chi(x) = R_c/R_0(x)$ of the curvature radii (or curvatures) can be obtained from (4) and (11):

$$\chi(x) = 1-\frac{R_c}{K}p''(x) = 1-\phi_1(u)\left[V_0(\alpha x)+\frac{V_3(u)}{V_1(u)}V_2(\alpha x)\right], \tag{26}$$

where the function

$$\phi_1(u) = \frac{V_1(u)}{V_0(u)V_1(u)+V_2(u)V_3(u)} = 2\frac{\cosh u \sin u+\sinh u \cos u}{\sinh 2u+\sin 2u} \tag{27}$$

characterizes the curvature ratio at the origin, and, like the functions $\chi_1(u)$ and $\chi_2(u)$, changes from unity to zero, when the u value changes from zero to infinity (Table 1). As is evident from (21) and (27), the function $\phi_1(u)$ is negative in the region between $u_1 = 2.365$ and $u_2 = \frac{7}{4}\pi$. In this region the curvature ratio

$$\chi_0 = \chi(0) = 1-\phi_1(u) \tag{28}$$

at the origin is greater than unity, i.e. the curvature of the glass fiber becomes greater than the coating curvature (Fig. 2). The maximum χ_0 value occurs at $u = \pi$ and is $\chi_0 = 1+1/\cosh\pi = 1.0864$. In the range $\pi < u < \frac{7}{4}\pi$ the increase in the coating compliance, leading to smaller u values, results in larger (not smaller!) curvatures of the glass fiber at the origin. As follows from the results of the analysis of the behavior of the function $p(x)$, this paradoxical situation is due to the redistribution of the interfacial load. Such a redistribution occurs at certain combinations of the coating compliance and the length of the curved area. Note that this phenomenon was observed experimentally (Marinis *et al.*, 1984) and analysd theoretically (Suhir, 1988b) earlier in connection with the mechanical behavior of external electrical leads in compliant-leaded surface-mounted electronic devices.

Although the function $\chi_0(u)$ oscillates with an increase in the u value, its amplitudes fade so rapidly that, in effect, only its first maximum at $u = \pi$ is appreciably different from unity (Fig. 2). Indeed, since all the extrema of this function occur in the region of relatively large u values ($u > 2$), eqn (28) can be replaced by the following approximate relationship:

$$\chi_0(u) = 1 - 2\,e^{-u}(\cos u + \sin u).$$

The maxima of this function are

$$\chi_{max} = 1 + 2\,e^{-m\pi}, \quad m = 1, 3, 5, \ldots$$

While the first maximum ($m = 1$) is equal to 1.0864, the second maximum ($m = 3$) is only 1.00016, i.e. very close to unity. Therefore, when the combination of the coating compliance and the length of the curved area is such that the u value is greater than $u_2 = \frac{7}{4}\pi = 5.50$, the change in the u value has a negligible effect on the glass fiber curvature at the origin.

Applying the same approach as we used earlier when analysing the behavior of the load function to the curvature ratio $\chi(x)$, we find that the extrema of this ratio occur at the locations $x = x_c$ which can be determined from the equation

$$V_1(u)V_3(\alpha x_c) - V_3(u)V_1(\alpha x_c) = 0,$$

or

$$\tanh u \tan\left(u\frac{x_c}{l}\right) - \tan u \tanh\left(u\frac{x_c}{l}\right) = 0, \tag{29}$$

and that the curvatures of the glass fiber and its coating have the same sign if the condition

$$V_3(u)V_0(\alpha x_c) - V_1(u)V_2(\alpha x_c) > 0 \tag{30}$$

is fulfilled. The numerical solution to the transcendental equation (29) is plotted in Fig. 2, and is illustrated by the plot in Fig. 3. It is shown in this figure how the $\alpha x_c = u(x_x/l)$ value can be obtained for the given u value. As is evident from the obtained solution, no additional internal extrema occur on the curvature ratio curve $\chi(x)$, if the u value is below $\frac{5}{4}\pi = 3.927$.

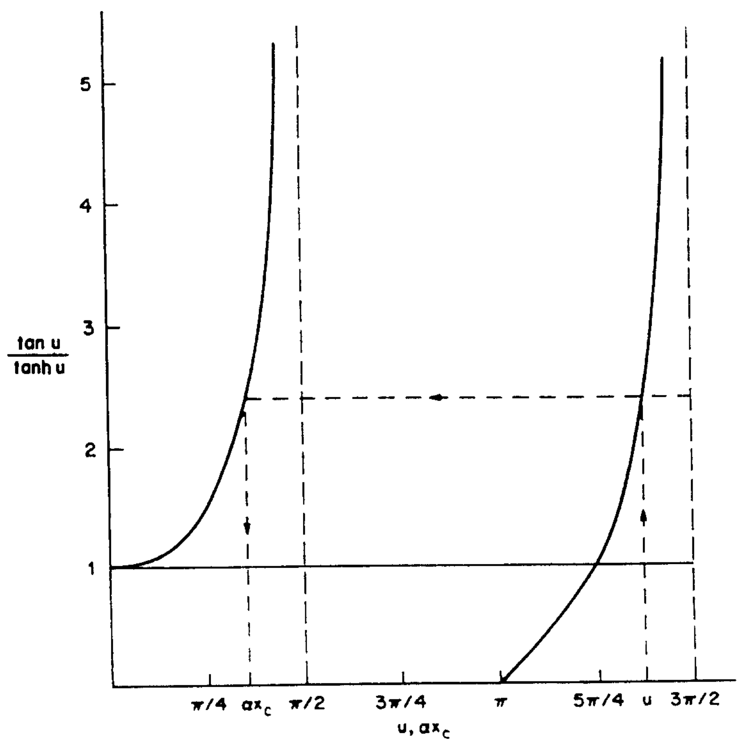

Fig. 3.

In this case the extremum at the origin is the only one, and, since the inequality (30) is fulfilled, the curvature of the glass fiber has the same sign as the coating curvature throughout the curved area. When the u value is large enough, and therefore the x_c/l ratio is not small either, eqn (29) can be simplified as follows:

$$\sin\left[u\left(1-\frac{x_c}{l}\right)\right] = 0. \tag{31}$$

Then we obtain:

$$\frac{x_c}{l} = 1-\frac{\pi}{u}. \tag{32}$$

Comparison of this approximate formula with the exact solution based on eqn (29) shows that this formula is accurate enough for $u > 4.3$. The comparison of eqn (32) with eqn (25) indicates that the external extrema of the load function are located closer to the ends of the curved area than the extrema of the curvature ratio. Substituting (32) into the condition (30), we conclude that the latter condition is fulfilled. This means that the sign of the glass fiber curvature at the locations $x = x_c$ is the same as the sign of the curvature of the coating.

For sufficiently large u values eqn (27) can be simplified as follows:

$$\chi(x) = 1-e^{-\alpha(l-x)}\left[\sin\alpha(l-x)+\cos\alpha(l-x)\right]. \tag{33}$$

As is evident from this formula, the curvature ratio can be greater than unity, if the expression in brackets is negative. Introducing the $x = x_c$ value from (32) into (33), we obtain: $\chi_c = 1+e^{-\pi} = 1.0432$. Note that this value is independent of the parameter u and its deviation from unity is twice as small as the deviation of the maximum value χ_0 of the curvatures ratio at the midpoint of the curved area. This, as has been shown earlier, takes place for $u = \pi$. The calculated χ_c values are plotted in Fig. 2.

NUMERICAL EXAMPLES

In the case of a *single-coated fiber*, when a 62.5 μm thick Borden Type 1 polymeric material ($E_1 = 5000$ psi, $\nu_1 = 0.495$, $r_1 = 125$ μm) is used to coat a glass fiber ($E_0 = 10.5\times10^6$ psi) with a radius $r_0 = 62.5$ μm, eqn (3) yields: $K = 178{,}690$ psi. From (7) we find: $\alpha = 4.341$ mm^{-1}. Using data from Fig. 2, we find that when the length of the curved area is smaller than

$$2l = 2\frac{u_1}{\alpha} = 2\times\frac{2.365}{4.341} = 1.09 \text{ mm}$$

(8.7 of the glass fiber diameter), the maximum curvature of the glass fiber is smaller than the curvature of the coating. If the length of the curved area is between 1.09 mm and

$$2l = 2\frac{u_2}{\alpha} = 2\times\frac{5.498}{4.341} = 2.53 \text{ mm}$$

(20.2 of the glass fiber diameter), then the maximum curvature of the glass fiber exceeds the coating curvature. The maximum curvature of the glass fiber takes place for a

$$2l = \frac{2\pi}{\alpha} = \frac{2\pi}{4.341} = 1.45 \text{ mm}$$

long curved area, and is by a factor of 1.0864 greater than the coating curvature. If the

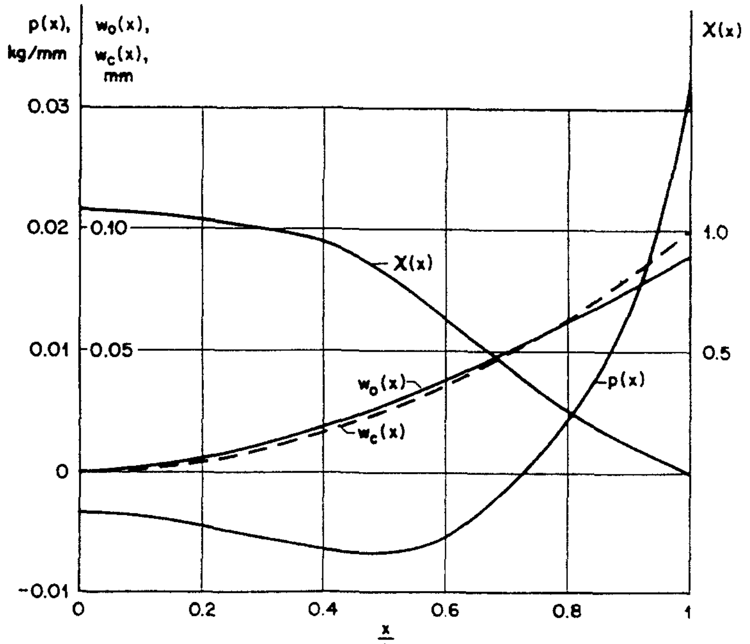

Fig. 4. The longitudinal distribution of the interfacial load $p(x)$, the deflections of the glass fiber $w_0(x)$, the coating $w_c(x)$, and the fiber-to-coating curvature ratio $\chi(x)$.

curved area is longer than 2.53 mm, the curvature of the glass fiber in its midpoint is not different from the curvature of the coating. If, for instance, the curved area is 5 mm long ($l = 2.5$ mm), then the u value is $u = 10.852$, and the maximum curvature ratio $\chi_c = 1.0432$ occurs at the points

$$\frac{x_c}{l} = 1 - \frac{\pi}{u} = 1 - \frac{\pi}{10.852} = 0.7105.$$

This corresponds to the distances of 0.724 mm from the ends of the curved area.

In the current AT&T *dual-coated fiber* design, the primary coating is about 30 μm thick, and the elastic constants of the material (silicone) are about $E_1 = 100$ psi and $\nu_1 = 0.5$. Then the calculated spring constant value, assuming absolutely rigid secondary coating, is about $K = 44{,}270$ psi, and therefore $\alpha = 3.062$ mm^{-1}. In this case the range of the fiber lengths which result in the midpoint curvatures of the glass fiber exceeding the coating curvature is between 1.54 and 3.59 mm. The curvatures of the glass fiber for the curved area lengths below 1.54 mm are smaller than the curvature of the coating. In the case of curved areas longer than 3.59 mm, the maximum curvature ratios occur at the locations defined by eqn (33). If, for instance, the curved area is 5 mm long, then the u value is $u = 7.655$, so that $x_c/l = 0.5896$. This corresponds to the distances of 1.126 mm from the ends of the curved area. The load function $p(x)$, the coating deflections $w_c(x)$, the glass fiber deflections $w_0(x)$, and the curvature ratios $\chi(x)$, calculated for the AT&T dual-coated fiber design, are shown in Fig. 4 for the coating radius $R_0 = 5$ mm and the length of the curved area of 2 mm.

CONCLUSION

A simple and easy-to-apply analytical stress model has been developed to evaluate the elastic curve of an optical glass fiber with a constant (measured or imposed) bend radius of its coating. We show that in order to predict the behavior of a glass fiber whose coating is subjected to bending one should compute first parameter u. This parameter depends, in addition to Young's modulus and diameter of the glass fiber itself, also on the length of the curved area, as well as Young's modulus and outer diameter of the (primary) coating. In the range $0 < u < 2.365$ (short curved areas and/or very compliant coatings), the maximum curvature of the glass fiber occurs at the midpoint of the curved area, never exceeds the curvature of the coating, and increases with an increase in the length of the curved area and the spring constant (stiffness) of the coating. If the calculated u value is greater than $7\pi/4 = 5.50$, the curvature of the glass fiber in the midportion of the glass-coating composite is practically the same as the coating curvature. In this case the maximum curvatures of the glass fiber are shifted towards the ends of the curved area and exceed the curvature of the coating by a factor of 1.043. These maxima are the closer to the ends of the curved area, the greater the u value is (i.e. the longer is the curved area and/or the greater is the spring constant of the coating). The magnitudes of these maxima are independent of the u value, i.e. do not change with the change in the length of the curved area and/or the coating compliance. If the combination of the length of the curved area and the coating compliance is such that u value falls within the range $2.365 < u < 5.50$, then the maximum curvature of the glass fiber is greater than the curvature of the coating. The most unfavorable combination of the length of the curved area and the coating compliance corresponds to $u = \pi$ and results in the curvature of the glass fiber at its midpoint exceeding by a factor of 1.086 the curvature of the coating. The deviation of the glass fiber curvature from the curvature of the coating is in this case twice as large as this a deviation in the case of a long curved area and/or a stiff coating ($u > 5.50$). In the range of the u values between π and $\frac{7}{4}\pi = 5.50$, the increase in the coating compliance (for the given length of the curved area) results in larger (not smaller!) curvatures of the glass fiber at its midpoint. This paradoxical situation is due to the redistribution of the interfacial load acting on the glass fiber at different combinations of the lengths of the curved areas and coating compliances.

REFERENCES

Arfken, G. (1985). *Mathematical Methods for Physicists*. Academic Press, Orlando, FL.

Marinis, T. F., Reinert, R. C. and Sherry, W. M. (1984). Impact of external lead design on the fracture of HIC-PWB assemblies subjected to bending. *Proc. 34th Electronic Component Conference*, New Orleans, LA, pp. 281–293. IEEE/CHMT.

Suhir, E. (1988a). Spring constant in the buckling of dual-coated optical fibers. *IEEE/OSA J. Lightwave Technol.* **6**(7), 1240–1244.

Suhir, E. (1988b). On a paradoxical phenomenon related to beams on elastic foundation: Could external compliant leads reduce the strength of a surface-mounted device? *ASME J. Appl. Mech.* **55**(10), 143–148.

Suhir, E. (1989). Interfacial stresses in bi-metal thermostats. *ASME J. Appl. Mech.* **56**(3), 657–660.

Timoshenko, S. P. and Young, D. H. (1965). *Theory of Structures*. McGraw-Hill, New York.

Vangheluwe, D. C. L. (1984). Exact calculations of the spring constant in the buckling of optical fibers. *Appl. Opt.* **23**(13), 2045–2046.